tor which is coupled through a resistance capacity filter to the power amplifiers.

Power Amplifier

The amplifier consists of a cathode coupled phase inverter driving a class A push-pull parallel power amplifier which gives an output of approximately 15 watts at 250 volts. This is sufficient to supply the power requirements of the synchronous motors. Correction for power factor is necessary when coupling these motors to the amplifier output transformer.

Calibration Signal Generator

The 1000 Hz sine wave calibration signal is derived from the 2000 pulses per second output of divider number three which is fed to a bi-stable multivibrator. The output of the multivibrator is a 1000 Hz square wave and this is coupled to a frequency selective amplifier employing a parallel T network. The output of this amplifier is essentially a pure 1000 Hz sine wave and is coupled through a cathode follower to provide a low impedance source of calibration signal.

Power Supply

Power supply to the crystal clock section is regulated electronically to maintain divider stability with line voltage variations.

The Discontinuity Capacitance and the Effective Position of a Shielded Open Circuit in a Coaxial Line

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Summary

The method described by Whinnery, Jamieson and Robbins in 1944 for the calculation of the coaxial line discontinuity capacitance has been applied to the coaxial open circuit, treated as a special case of a coaxial line with a stepped inner conductor.

List of Symbols

C_d = step capacitance, F/m = 10^{10} pF/cm.

v = velocity of propagation in vacuum = $= 2.997925 \times 10^8$ m/s.

f = frequency, Hz.

ω = angular frequency, rad/s.

Z_0 = characteristic impedance of loss-free coaxial line, ohms.

β = phase change constant of loss-free coaxial line, rad/m.

λ = wavelength of the fundamental wave (TEM) in a coaxial line, m.

r_a, r_b = respective radii of inner and outer conductors of a coaxial line, m.

l = the effective position of the equivalent perfect open circuit, measured from the end face of the inner conductor, m.

R = l/b, the effective position of the equivalent perfect open circuit, normalized to the annulus, $b = r_b - r_a$.

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Manuscript received by The Institution April 26, 1966.
Revised manuscript received August 12, 1966.
U.D.C. number 621.372.2 : 621.3.012.4.

1. Introduction

The method for determining the effects of discontinuities in cavity resonators developed by Hahn[1] has been used by Whinnery, Jamieson and Robbins to compute the effects of discontinuities in coaxial lines[2]. The principle of the method is to take into account all the evanescent modes generated by the sudden discontinuity in the transmission line, and adjust the amplitudes and phases of all these modes in such a way that the combined effect of these waves leads to electric and magnetic fields which satisfy the given boundary conditions. It was also shown[2] that the effect of a step-discontinuity in a transmission line is an equivalent shunt capacitance placed at the plane of the discontinuity.

The values of this equivalent capacitance for different discontinuities have been given by Whinnery, Jamieson and Robbins in the form of graphs[2]. Recent investigation has shown that the original charts were in error by up to 5 per cent. The need for more accurate values of "step capacitances" made it necessary to program for a digital computer the method given by the original authors. A description of the method of programming and the step capacitances for various discontinuities are given elsewhere[3].

1. Hahn, W. C., "A New Method for the Calculation of Cavity Resonators", *J. Appl. Phys.*, Vol. **12**, January, 1941, p. 62.
2. Whinnery, J. R., Jamieson, H. W. and Robbins, T. E., "Coaxial Line Discontinuities", *Proc. Instn Radio Engrs*, Vol. **32**, November 1944, p. 695.
3. Somlo, P. I., "The Computation of Coaxial Line Step Capacitances", *Trans. I.E.E.E.*, Vol. MTT-15, January 1967.

This paper deals with the special case of the stepped inner conductor in a coaxial line, in which the radius of the inner conductor is abruptly reduced to zero, thus resulting in a shielded coaxial open circuit.

2. Results

The computed values of the step capacitance shunting the line at the discontinuity (open circuit), and the effective position of the open circuit are given in table 1, and the latter in graphical form in fig. 1.

From the expression for the characteristic impedance of a coaxial line $Z_0 = 2 \times 10^{-7}\, \nu \ln(r_b/r_a)$ the annulus b is obtained :

$$b = r_b(1 - \exp(-Z_0/2 \times 10^{-7}\nu)) \qquad (3)$$

Substituting $\lambda = \nu/f$ in eq. (2) and using eq. (3) the ratio $R = l/b$ is obtained :

$$R = \frac{\lambda}{r_b} \frac{\tan^{-1}[4\pi^2 \nu Z_0 C_d (r_b/\lambda)]}{2\pi[1 - \exp(-Z_0/2 \times 10^{-7}\nu)]} \qquad (4)$$

Table 1.

$r_b/\lambda =$	0.0		0.1		0.2		0.3		0.35		0.38274	
Z_0	C_d	R	C_d	R	C_d	R	C_d	R	C_d	R	C_d	R
5	147.48	1.7361	150.36	1.7653	161.10	1.8739	194.28	2.2023	257.73	2.7867	∞	8.1637
10	110.61	1.3563	113.30	1.3811	123.41	1.4723	154.89	1.7365	215.47	2.1570	∞	4.2520
25	65.306	0.9020	67.344	0.9181	75.047	0.9755	99.280	1.1240	145.82	1.3149	∞	1.9158
50	36.242	0.6034	37.408	0.6130	41.780	0.6460	55.122	0.7265	78.867	0.8212	∞	1.1547
75	22.732	0.4499	23.392	0.4565	25.834	0.4794	32.952	0.5353	44.510	0.6019	∞	0.9152
100	15.170	0.3522	15.559	0.3572	16.979	0.3748	20.982	0.4187	27.088	0.4724	∞	0.8051
300	1.7417	0.0991	1.7759	0.1009	1.9013	0.1075	2.2577	0.1261	2.8015	0.1534	∞	0.6576

C_d is the discontinuity capacitance of the shielded open circuit in a coaxial line in fF/cm* of circumference of the outer conductor ($2\pi r_b$), and $R(r_b - r_a)$ is the distance in cm of the equivalent perfect open circuit beyond the end of the inner conductor. C_d and R are functions of r_b/λ and the characteristic impedance of the coaxial line, Z_0 (ohm).

* 1 fF (femto Farad) = 1 mpF = 10^{-15} F.

Because the value of step capacitance is proportional to the size of line in which it occurs, it is convenient to normalize it to the line dimensions. Following the method used by Whinnery, Jamieson and Robbins[2], the values given are normalized to the circumference of the outer conductor. An alternative method is to express the value of step capacitance as an equivalent length of coaxial line terminated by a perfect open circuit, i.e. one having no fringing fields. This equivalent line has the same characteristic impedance as the original coaxial line and, at a given frequency, its input admittance is equal to the admittance of the calculated step capacitance.

$$Y_{in} = j/Z_0 \tan(\beta l) ; \quad Y_c = j\omega 2\pi r_b C_d$$
$$Y_{in} = Y_c \qquad (1)$$

From eq. (1)

$$l = 1/\beta \tan^{-1}(\omega 2\pi r_b C_d Z_0) = \nu/\omega \tan^{-1}(\omega 2\pi r_b C_d Z_0) \qquad (2)$$

Because C_d is proportional to the size of the line, for a universally valid representation of the position of the effective open circuit l should be normalized to line size. It was found to be convenient to normalize l to the annulus $b = r_b - r_a$, because for small values of Z_0, $R = l/b$ varies quasi-logarithmically, leading to an easy representation on a semi-logarithmic chart (fig. 1).

R is a dimensionless quantity, locating the position of the effective open circuit expressed in units of the annulus $r_b - r_a$, being a function of the characteristic impedance of the line, where the parameter r_b/λ indicates the frequency dependence.

It is worth noting that the top curve in fig. 1 corresponds to a frequency at which the first higher mode (TM_{01}) begins propagating, resulting in $C_d = \infty$. Thus an effective short circuit is produced at the end face of the inner conductor, which is equivalent to an effective open circuit located at $\lambda/4$ distance from the physical end of the inner conductor.

However, it should be pointed out that for a given Z_0, values of R corresponding to the parameter r_b/λ greater than that at which the coaxial line starts supporting the TE_{11} mode are mainly of academic interest because R is defined in terms of a line supporting the TEM mode only. Above this limiting value of r_b/λ the curves for R are dashed in fig. 1 and may be used only if it is assured that only the TEM mode is present in the coaxial line. As a guide, to indicate the range of the parameter r_b/λ at which the TE_{11} mode starts propagating in the coaxial section of the line, at $Z_0 = 0$, $r_b/\lambda = 0.159$ and at

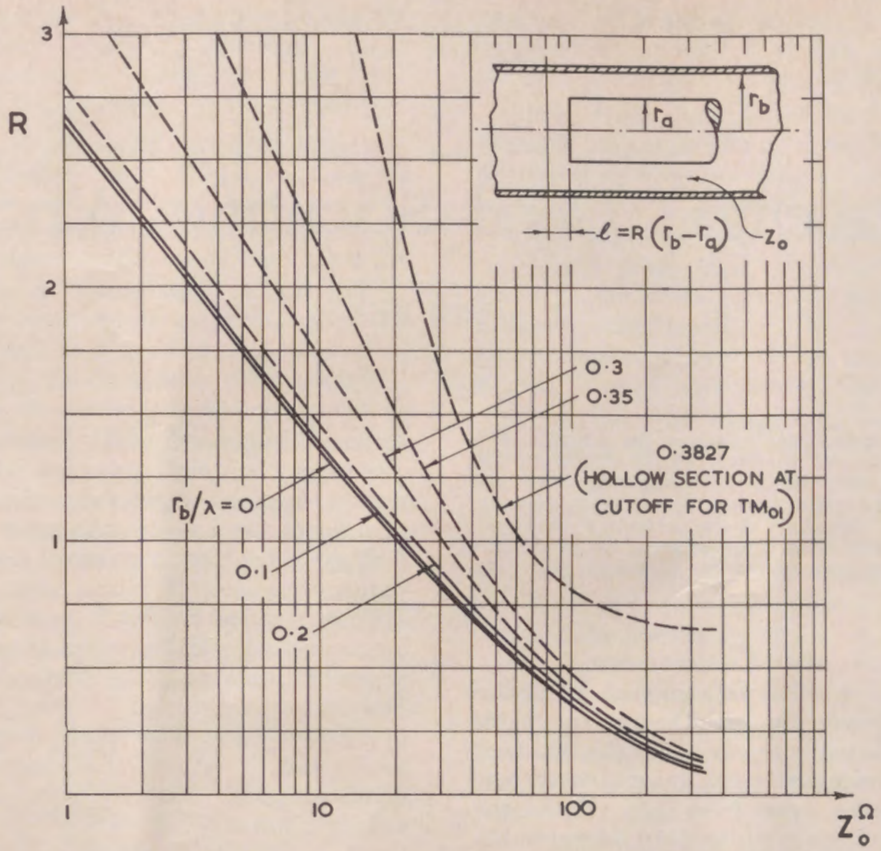

Figure 1.—Plot of R versus Z_o and r_b/λ, where $R(r_b - r_a)$ is the distance of the equivalent perfect open circuit beyond the end of the inner conductor of a coaxial line having a characteristic impedance Z_o (ohm), operated at a (TEM) wavelength of λ. The continuous portions of the curves (if any) represent a condition for TEM propagation only in the coaxial line.

$Z_0 = \infty$, $r_b/\lambda = 0.318$. The properties of higher modes in coaxial lines are readily available[4].

3. Calculation of the Step Capacitance

The details of the programming were given elsewhere[3], and are not repeated here. The only difference from the described calculation[2] is that eq. (8) is changed to $J_0(k_{Bn}r_3) = 0$, and that eq. (6) is changed to $G_{Bn} = 0$.* These changes are the result of dealing with the higher modes of propagation in hollow pipes instead of concentric cylinders.

The accuracy of the results is determined by the accuracy of the computed values of step capacitance, which are believed to be accurate to five significant digits. Details on accuracy are given in the literature[3].

4. Conclusion

Values of coaxial line step capacitances have been calculated using the "mode matching" principle described by Whinnery, Jamieson and Robbins in 1944, for the special case of the coaxial line shielded open circuit. This is the limiting case when the radius of the inner conductor is reduced to zero from a given transverse plane onwards. For transmission line calculations, or laboratory measurements, it is convenient to regard the open circuit discontinuity capacitance as equivalent to a lengthening of the line, leading to an effective position of a perfect open circuit. The position of the effective open circuit is given in fig. 1 as a function of Z_0 for different values of λ and line size.

4. Marcuvitz, N., "Waveguide Handbook", McGraw-Hill Book Company Inc., New York, 1951, p. 77.

*For the definitions of symbols k_{Bn}, r_3 and G_{Bn}, the reader is referred to reference 2.